

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Engineering Mathematics-II

Subject Code: 4TE02EMT1

Semester: II

Time: 10:30 To 1:30

Branch: B.Tech(All)

Date: 19/11/2015

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) A square matrix A is called orthogonal if
 (a) $AA^{-1} = I$ (b) $A^2 = A$ (c) $A^T = A^{-1}$ (d) $A^2 = I$
- b) A $n \times n$ Non-Homogeneous system of equations $AX = B$ is given. If $\rho(A) = \rho(A : B) = n$ then the system has
 (a) No solutions (b) Unique solutions
 (c) Infinite solution (d) None of these
- c) The rank of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is
 (a) 1 (b) 2 (c) 3 (d) -2
- d) The Sum of the eigenvalues of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is
 (a) 1 (b) 4 (c) 2 (d) 5
- e) Find the value of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & 3 \\ 1 & 0 & 0 \end{vmatrix} = \underline{\hspace{2cm}}$
 (a) 1 (b) 12 (c) -2 (d) 0
- f) A square matrix A is called Singular if
 (a) $|A| = 0$ (b) $A^2 = A$ (c) $AA^T = I$ (d) $|A| \neq 0$
- g) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = \underline{\hspace{2cm}}$
 (a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$



- h) $\int_0^{\pi/2} \cos^4 x \, dx = \underline{\hspace{2cm}}$
 (a) 0 (b) 1 (c) $\frac{3\pi}{16}$ (d) $\frac{8\pi}{3}$
- i) $\int_0^1 \int_0^x dy \, dx = \underline{\hspace{2cm}}$
 (a) $\frac{1}{2}$ (b) -1 (c) 0 (d) y
- j) The value of $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$ for $m \neq \pm n$ is
 (a) 0 (b) π (c) $\frac{\pi}{2}$ (d) 2π
- k) Angle between the vectors $2i + 2j - k$ and $6i - 3j + 2k$ is
 (a) $\cos^{-1}\left(\frac{4}{11}\right)$ (b) $\cos^{-1}\left(\frac{4}{21}\right)$ (c) $\sin^{-1}\left(\frac{4}{11}\right)$ (d) $\cos^{-1}\left(\frac{4}{21}\right)$
- l) $\text{div curl } \vec{V} = \underline{\hspace{2cm}}$
 (a) 0 (b) 1 (c) $\vec{0}$ (d) \vec{V}
- m) A vector \vec{F} is said to be irrotational if
 (a) $\nabla \times \vec{F} = 0$ (b) $\nabla \cdot \vec{F} = 0$ (c) $\nabla \vec{F} = 0$ (d) None of these
- n) If $\begin{bmatrix} x & 2 \\ 3 & 1 \end{bmatrix}$ is a singular matrix then $x = \underline{\hspace{2cm}}$
 (a) 1 (b) 6 (c) 2 (d) -6

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$ by using determinant method. (05)

b) Evaluate: $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$ (05)

c) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ to the normal form and find its rank. (04)



Q-3 Attempt all questions

a) Find the eigenvalues & eigenvectors of a matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (05)

b) Solve the following system of equations by Cramer's rule: (05)
 $x + 2y - z = 3; \quad x + y + 2z = 9; \quad 2x + y - z = 2$

c) Determine $\int_0^1 \ln x \, dx$ converge or diverges. (04)

Q-4 Attempt all questions

a) Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. (05)

b) Find the inverse of the following matrix by using elementary transformation (05)

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & -2 & 1 & 6 \end{bmatrix}$$

c) Solve: $\frac{dy}{dx} + y \tan x = \sin 2x, \quad y(0) = 1$ (04)

Q-5 Attempt all questions

a) Obtain Row echelon & Reduced row echelon form of the following matrix: (05)

$$A = \begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

b) Solve: $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$ (05)

c) Find the directional derivatives of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. (04)

Q-6 Attempt all questions

a) Evaluate $\int_C \bar{F} \, d\bar{r}$ where $\bar{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the rectangle in the xy -plane bounded by $y = 0, x = a, y = b, x = 0$. (05)



b) Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = 18z\hat{i} - 12y\hat{j} + 3y\hat{k}$ and S is the part of the plane **(05)**

$2x + 3y + 6z = 12$ in the first octant.

c) Solve the system of equation by Gauss-Elimination method. **(04)**

$$2x + 2y + 2z = 0$$

$$-2x + 5y + 2z = 1$$

$$8x + y + 4z = -1$$

Q-7 Attempt all questions

a) Change the order of integration and evaluate $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx dy$. **(05)**

b) Solve: $\left(x + \frac{ay}{x^2 + y^2}\right)dx + \left(y - \frac{ax}{x^2 + y^2}\right)dy = 0$ **(05)**

c) Evaluate: $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$ **(04)**

Q-8 Attempt all questions

a) Verify Green's theorem for $\oint_C [(x - y)dx + 3xy dy]$ where C is the boundary **(07)**

of the region bounded by the parabolas $x^2 = 4y$ and $y^2 = 4x$.

b) State Cayley-Hamilton theorem and Find the characteristic equation for the **(07)**

matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also find the matrix represented

by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

